

# **PT326 PROCESS TRAINER**

# PT 326 Process Trainer

## 1. Description of the Apparatus

The **PT 326 Process Trainer** models common industrial situations in which temperature control is required in the presence of transport delays and transfer lags.

The process contained in the PT 326 involves air that is drawn from the atmosphere by a centrifugal blower, and is heated as it passes over a heater grid before being released into the atmosphere through a duct. The control objective is to maintain the temperature of the air at a desired level. Temperature control is achieved by varying the electrical power supplied to the heater grid. The air temperature may be sensed by using a bead thermistor placed in the flow at any of three positions along the duct. The spatial separation between the thermistor and the heater coil introduces a transport delay into the system. Figure 1 shows the front panel of the PT 326 apparatus.

The mass flow of air through the duct can be adjusted by setting the opening of the throttle. The temperature sensor can be placed at any of the three locations marked P1, P2 and P3. The leads of the temperature sensor are connected to the terminals marked “detector”. The variation in the resistance of the temperature sensor is converted to a voltage variation by a bridge circuit, and is available at terminal Y. The output of the temperature sensor can be used for feedback by connecting terminals X and Y.

The desired temperature may be indicated by adjusting the knob labeled “set value”. An additional step change in the set value can be commanded by throwing the switch marked “internal”. An external reference input can be provided at terminal D labeled “external”. The difference between the reference input + set value and the feedback signal is available at terminal B, and can be applied to the heater power supply through a gain that can be adjusted by using the knob marked “proportional band”. A proportional band of PB % corresponds to a gain factor of  $100/PB$ . The signal at B is available at terminal A after amplification by the gain factor.

The signal at A may be applied directly to the heater power supply by using the switch S to select “continuous control”. Alternatively, the signal may be used in an on-off controller by using the switch S to select “two-step control”. The on-off controller commands maximum heater power when on, and zero heater power when off. The hysteresis in the on-off controller can be adjusted by using the knob marked “overlap” while the maximum heater power to be supplied can be adjusted by using the knob marked “max heater power”. The signal driving the heater power supply can be monitored at terminal C.

## 2. System Modeling.

The physical principle which governs the behavior of the thermal process in the PT326 apparatus is the balance of heat energy. The rate at which heat accumulates in a fixed volume V enclosing the heater is,

$$q_a = q + q_i - q_o - q_t \quad (1)$$

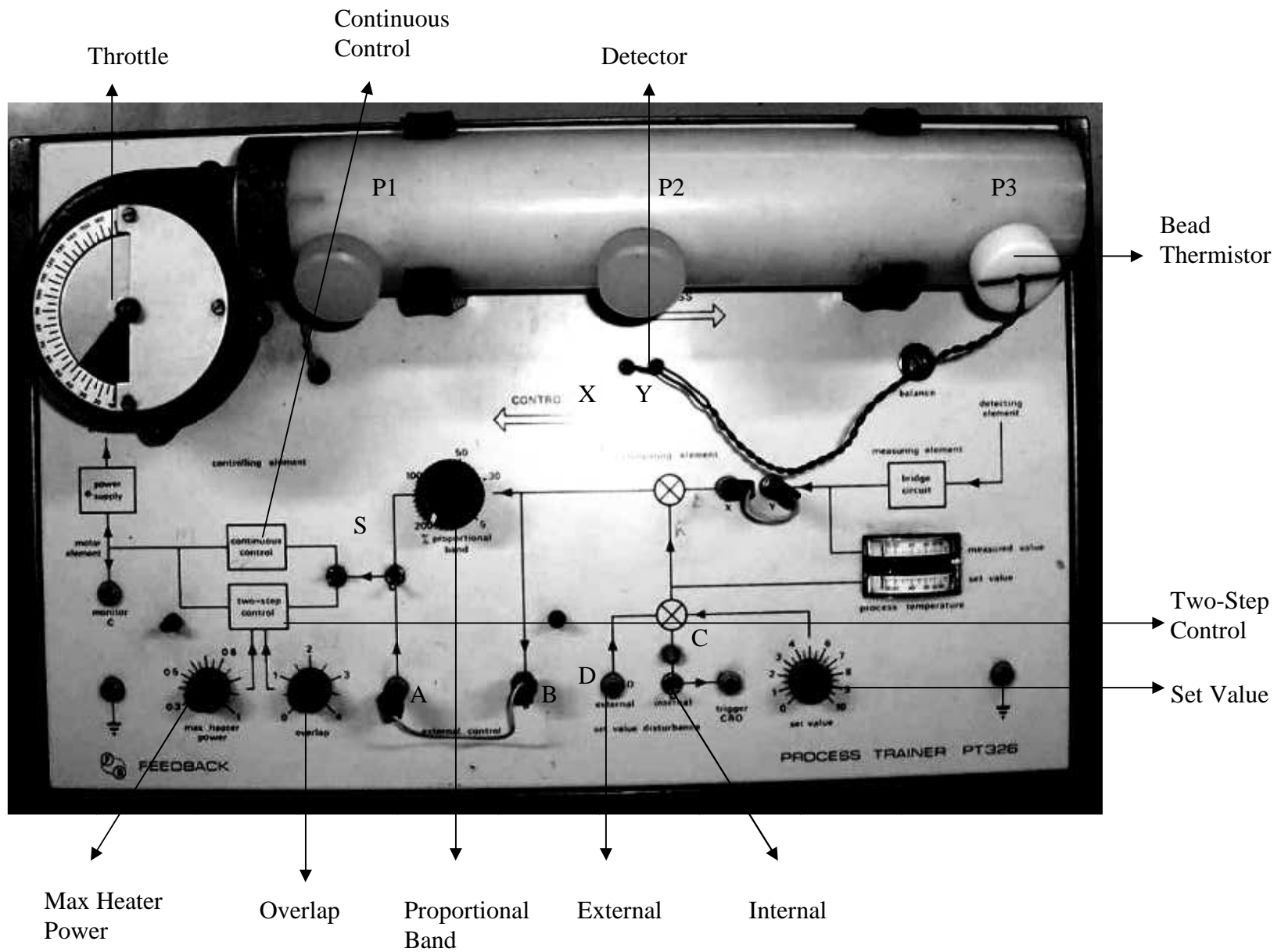
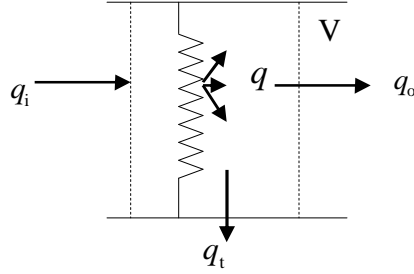


Figure 1: Front panel of the PT326 apparatus

where  $q$  is the rate at which heat is supplied by the heater,  $q_i$  is the rate at which heat is carried into the volume  $V$  by the coming air,  $q_o$  is the rate at which heat is carried out of the volume  $V$  by the outgoing air and  $q_t$  is the heat lost from the volume  $V$  to the surroundings by radiation and conduction. Figure 2 below depicts the volume  $V$ .



**Figure 2: Heat transfer from the volume  $V$**

The accumulation of heat in the volume  $V$  causes the temperature  $T$  of air in  $V$  to rise. Assuming a uniform temperature in the volume  $V$ , the rate of heat accumulation is given as,

$$q_a = C \frac{dT}{dt}, \quad (2)$$

where  $C$  is the heat capacity of the air occupying the volume  $V$ .

The rates at which the air flow carries heat in and out of the volume  $V$  are given respectively by  $q_i = C \dot{m} T_a$  and  $q_o = C \dot{m} T$ , where  $C$  is the specific heat capacity of air,  $\dot{m}$  is the mass flow rate and  $T_a$  is the ambient air temperature. Assuming that the rise in air temperature  $\Delta T = T - T_a$  is small, the rate  $q_t$  at which heat is lost from the volume  $V$  is proportional to the temperature rise  $\Delta T$ . Thus,

$$q_i - q_o - q_t = -\frac{1}{R} \Delta T, \quad (3)$$

where  $-\frac{1}{R}$  is a proportionality constant called the thermal resistance. Equations (1), (2) and (3) yield,

$$C \frac{d\Delta T}{dt} + \frac{1}{R} \Delta T = q_i. \quad (4)$$

Taking Laplace transform yields

$$\frac{\Delta T(s)}{q_i(s)} = \frac{k_1}{\tau s + 1}, \quad (5)$$

where  $k_1 = \frac{1}{R}$  and  $\tau = RC$  is the time constant. On assuming the heater heat supply rate to be proportional to the heater input voltage  $V_i$ , (5) yields the transfer function between the heater input voltage and the temperature rise as,

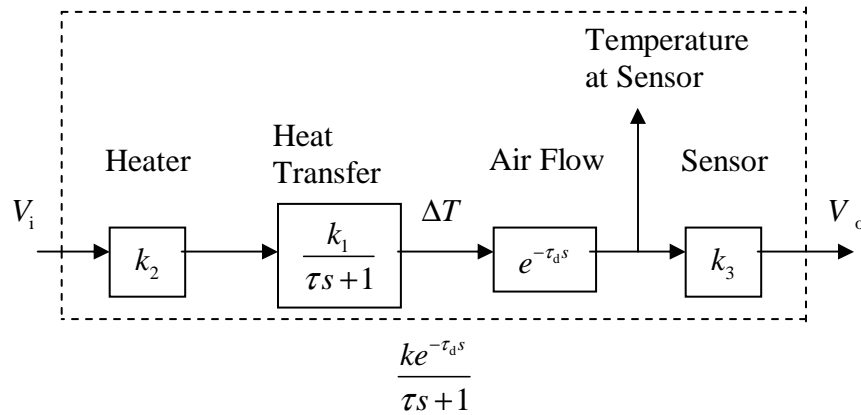
$$\frac{\Delta T(s)}{V_i(s)} = \frac{k_1 k_2}{\tau s + 1}, \quad (6)$$

where  $k_2$  is the proportionality constant between  $q$  and  $V_i$ .

In equation (6),  $\Delta T$  represents the increase in temperature of the air contained in the volume  $V$  considered above. The temperature sensor produces a voltage  $V_o$  that is proportional to  $\Delta T$ , that is  $V_o = k_3 \Delta T$ . However, since the sensor is physically located at a certain distance from the heat source, the sensor output responds to a temperature change with a pure time delay  $\tau_d$ , which is the time taken by the flowing heated air to cover the distance between the heater and the sensor. Thus the transfer function between the heater input voltage and the sensor output voltage is,

$$\frac{V_o(s)}{V_i(s)} = \frac{k e^{-\tau_d s}}{\tau s + 1}, \quad (7)$$

where  $k = k_1 k_2 k_3$  is the DC gain of the system. The term  $e^{-\tau_d s}$  in (6) arising due to fluid transport is called a *transport delay*, while the term  $(\tau s + 1)^{-1}$  arising due to the heat transfer dynamics is called *transfer delay*. Figure 3 below illustrates equations (5)-(7).



**Figure 3: Block diagram of the PT326 process**

**Question:** How are the parameters  $k$ ,  $\tau_d$  and  $\tau$  affected by the sensor location and throttle opening?

### 3. System Step Response

The output of the temperature sensor  $V_o$  and the heater input voltage are related by the first-order transfer function (7) for small temperature changes from the ambient. The transfer function in (7) is characterized by two parameters, namely, the DC gain  $K$  and the time constant  $\tau$ . Both of these parameters can be determined from the response of the temperature to a step increase in the heater input voltage from a state of thermal equilibrium.

To find an expression for the unit step response of the temperature variation, recall that the Laplace transform of a unit step input is  $\frac{1}{s}$ . Hence the Laplace transform of the response of the temperature variation  $\Delta V_o$  to an increase of 1V in the heater input voltage is

$$\Delta V_o(s) = \frac{Ke^{-\tau_d s}}{s(\tau s + 1)}, \quad (8)$$

Taking inverse Laplace transform yields

$$\Delta V_o(t) = K(1 - e^{-(t-\tau_d)/\tau}), \quad (9)$$

We notice that  $\Delta V_o$ , the change in temperature from the initial equilibrium value, converges to the value  $K$ . Thus the DC gain  $K$  is simply the ratio of the steady state change in the output to the steady state value of the input. We also observe that, at  $t = \tau + \tau_d$ , the value of the output is  $\Delta V_o(\tau) = K(1 - e^{-1}) = 0.6321K$ . Thus the time constant of the system is the time that it takes for the output to change by 63.21 % of the steady state change. Both the parameters  $K$  and  $\tau$  can thus be determined experimentally from the step response.

#### Experiment 1:

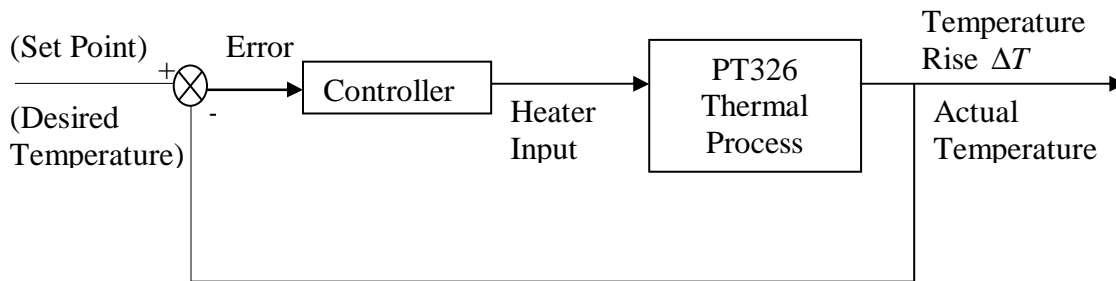
In this experiment, you will give step inputs to the open-loop system to determine the nature of the transfer function between the set value and the output of the temperature sensor.

Set the proportional band to 100% and the set value to 2. Use the appropriate switches to select "continuous control" and "proportional band". Perform this experiment by placing the sensor at each of the three locations P1, P2 and P3. For each sensor location, repeat the experiment at the three values 40, 65 and 90 of throttle setting. For every combination of the sensor position and throttle setting, give a set-value disturbance by throwing the switch marked "internal". This gives a step input voltage of a certain fixed magnitude, which can be seen on

the oscilloscope at the terminal marked ``trigger CRO". The output of the temperature sensor can be seen at the terminal marked ``Y". By observing both signals on the oscilloscope, determine the DC gain, transport lag, and time constant of the open-loop transfer function between the set value and the sensor output.

#### 4. On-Off Control.

A typical control objective in thermal systems is to maintain the temperature of some component at a user specified value called the set point. Figure 4 below depicts a closed-loop system designed to maintain the output temperature of the PT326 apparatus at a desired set point.

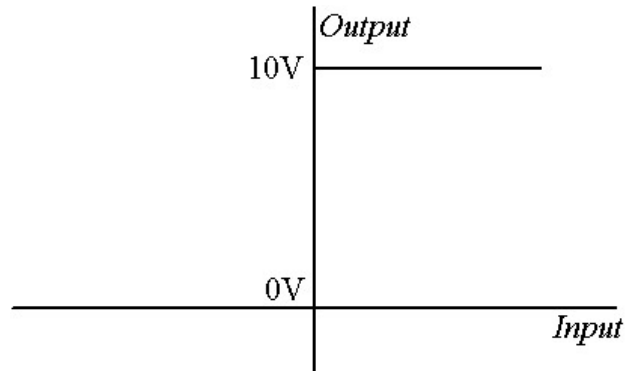


**Figure 4: Closed-loop control of a thermal system**

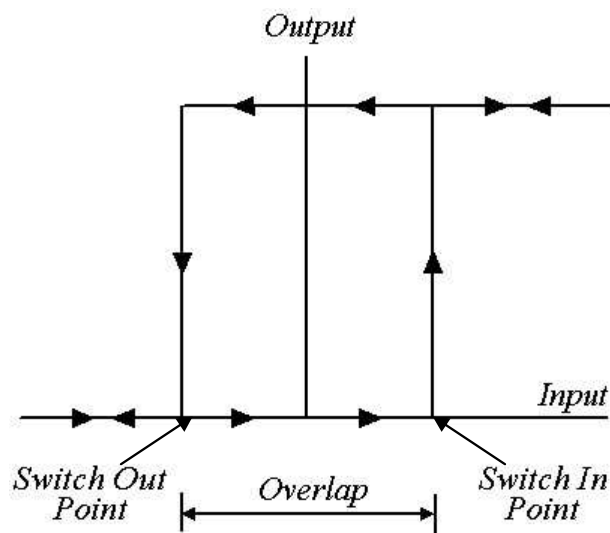
Loosely speaking, the controller uses the error signal between the desired temperature and actual temperature to manipulate the heater input voltage in such a way that the actual temperature converges to the set point value.

A cost effective means of implementing a controller in thermal systems is a relay, which gives full heater input voltage when the error signal is positive and gives zero heater input voltage when the error signal is negative. Such a controller is called an on-off controller. Figure 5 below shows the input-output characteristic of an on-off controller.

Practical relays suffer from hysteresis, where, for decreasing input, the relay output switches off at a lower value of the input than the value at which the relay output switches to its maximum when the input is increasing. Figure 6 below shows the input-output characteristics of a relay with hysteresis.



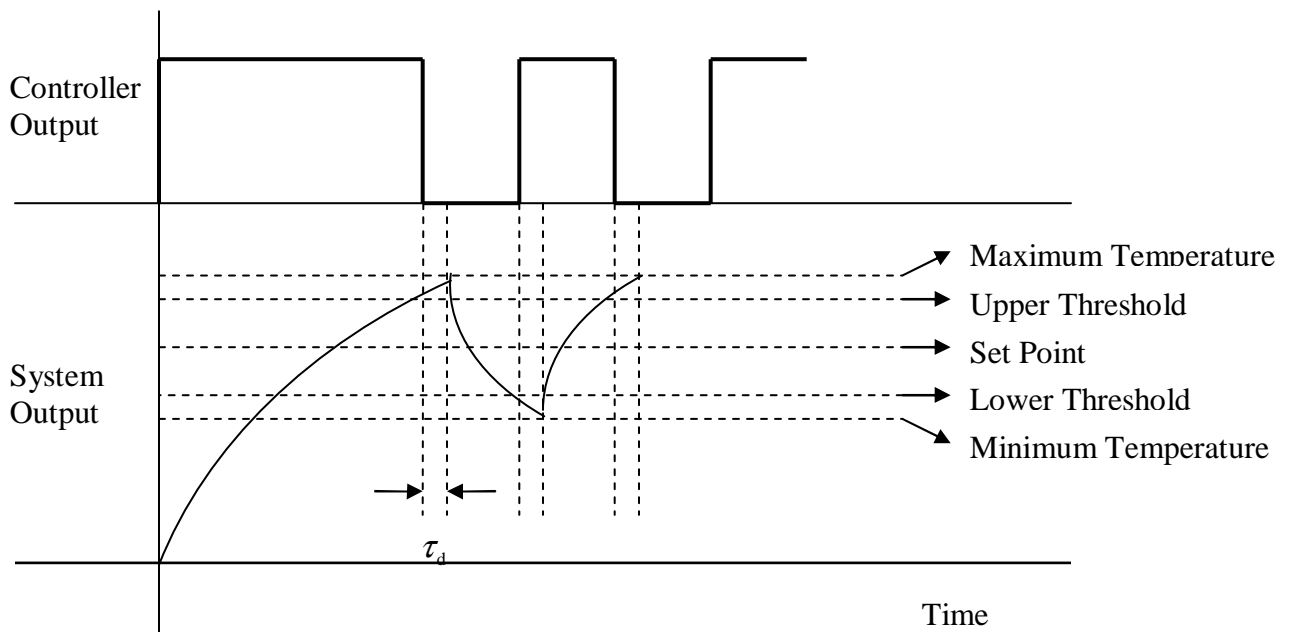
**Figure 5: Input-output characteristics of an on-off controller**



**Figure 6: Input-output characteristics of a relay with hysteresis**

Figure 7 below shows a typical closed-loop response of a first-order system under on-off control with hysteresis.





**Figure 7: Closed-loop response under on-off control**

**Experiment 2:**

In this experiment, you will investigate how the open-loop behavior influences the behavior of the closed-loop system under on-off control.

Place the sensor at the middle location. Set the proportional band to 100%, the set value to 2, throttle setting to 40, and overlap to 0. Select “two-step control” and switch off the internal set-value disturbance. Close the loop by connecting the terminals “X” and “Y”. Use the oscilloscope to observe the heater input voltage at terminal “C”, and the temperature sensor output at terminal “Y”. From the oscilloscope, measure the threshold value of sensor output below which the heater turns on, threshold value of sensor output above which the heater turns off, maximum and minimum values of the sensor output voltage, maximum and minimum values of the heater input voltage, cycle time, heater-on time and heater-off time in each cycle. Repeat the above steps by setting the overlap to 0.5 and 1.

## 5. Proportional Control.

On-off controllers, though inexpensive to implement, suffer from the disadvantage that they lead to cycling or “hunting” around the desired set point, and thus provide less exact control.

An alternative to an on-off controller is a proportional controller, whose output is proportional to the error between the set point and the actual measured output. Thus, when a proportional controller is used in the closed-loop system in Figure 5, the heater input voltage is related to the error  $e(s) = r(s) - V_o(s)$  by

$$V_i(s) = K_p(r(s) - V_o(s)), \quad (10)$$

where  $K_p$  is the proportional gain of the controller.

**Question:** Use (10) along with (7) to show that in the absence of transport delay, the closed-loop system is also a first-order system with DC gain

$$K_{CL} = \frac{K_p G}{1 + K_p G}, \quad (11)$$

and time constant

$$\tau_{cl} = \frac{\tau}{1 + K_p K}. \quad (12)$$

Equation (12) implies that the closed-loop response becomes faster as the proportional gain increases. Equation (11) implies that the DC gain approaches 1 as the proportional gain  $K_p$  increases. In other words, the steady-state value of the temperature approaches the set point as  $K_p$  increases. Thus, the closed-loop system responds faster and more accurately to constant set point commands as  $K_p$  increases. However, for no value of the gain  $K_p$  is the response completely accurate. This can be physically explained by observing that, if the error is zero, then the heater voltage commanded by the controller is zero, causing the air to start cooling immediately. In other words a proportional controller cannot maintain a state of zero error (unless the set point equals the room temperature!).

## 6: Frequency Response

The closed loop system shown in Figure 4 contains a pure delay. Such a system typically exhibits instability for sufficiently large values of the proportional gain. This can be understood by the following argument.

A phase lag of 180 degrees is added to the sensor output voltage as it is fed back to the heater input voltage with a negative sign. The transport lag and the transfer lag add a further phase lag to the heater input voltage. Because the phase lag due to the transport delay increases indefinitely with frequency, there exists a frequency at which the phase lag added by the transfer and transport lags is 180 degrees. This frequency is called the *phase crossover frequency*. Thus, at the phase crossover frequency, the total phase lag accumulated as a signal traverses the closed loop in Figure 4 is 360 degrees. If the proportional gain is such that the product of the gains around the loop is also unity, then a sinusoid at the phase crossover frequency can pass around the loop intact. In other words, the closed-loop system can exhibit sustained oscillations at the phase crossover frequency, indicating the onset of closed-loop instability.

It is clear from the argument above that the closed-loop system becomes unstable when the proportional gain equals the reciprocal of the gain of the open-loop transfer function (7) at the phase crossover frequency. This reciprocal is the *gain margin* of the system. In decibels, the gain margin is the negative of the gain of the transfer function (7) in decibels at the phase crossover frequency. The gain margin is the smallest value of the proportional gain above which the closed-loop system is unstable.

The gain margin of the thermal process in PT326 can be determined by experimentally generating the frequency response of the process. This is done by applying sinusoidal signals of different constant frequencies at the heater input voltage and observing the sensor output voltage. At each input frequency, the sensor output will be sinusoidal at the same input frequency. However, the output will be attenuated (or amplified) and suffer a phase lag when compared to the input. The ratios of the amplitudes of the output to the input is the gain of the system at the input frequency, while the phase lag between the output and the input is called the phase of the system at the input frequency. The gain and the phase of the system are measured experimentally at different input frequencies and plotted as a function of the frequency. The phase crossover frequency and is determined from the phase plot. The gain margin is then the reciprocal of the gain of the system at the phase crossover frequency.

**Experiment 3:** In this experiment, you will experimentally obtain the open-loop frequency response data for a thermal system and use it to predict the onset of closed-loop instability.

**Part I:** Open-loop frequency response

In this part, you will obtain frequency response plots for the open-loop system. Adjust “Set Value” to 35 degrees and blower inlet (throttle) to 40 degrees. Place the sensor in position P3. Select “Continuous Control” and “External Control”. Use a function generator to give a 2V peak-to-peak sinusoidal input at terminal A. Use an oscilloscope to compare the input signal and the sensor output (at Y). Vary the input frequency from 0.1 Hz to 3Hz, and measure the amplification and phase shift between the output and input at each frequency. Use your readings to draw Bode plots of the system. Find the phase crossover frequency and the corresponding magnitude from your Bode plots. Calculate the gain margin of the system.

## **Part II:** Closed-loop stability

Use the same settings for “Set Value” and blower inlet as above. Make connections such that the loop is closed with the proportional controller in the loop. By observing the output (in the absence of any external input), find the value of gain at which the closed-loop system becomes unstable. Compare this value with the critical value of gain calculated from the gain at phase crossover frequency.

**Question:** Find the value of gain at which the closed-loop system becomes unstable when the sensor is placed in position 2. Are the values for positions 2 and 3 same? Why?

Theory predicts that a negative feedback closed-loop consisting of a stable first-order system and a proportional gain is always stable. In this experiment, however, you observed that, for a sufficiently large gain, the closed loop is unstable even though the basic heat transfer process in the loop has first order dynamics. The instability of the closed loop is caused by the presence of the pure delay in the form of transport lag. This experiment thus highlights the danger that excessive delays and phase lags in the loop pose for stability. Delays and phase lags can also be compounded by higher-order dynamics, sampling, or computations.