Chapter 1

Force on a surface

The goal is to calculate the force on a surface due to hydrostatic pressure.

We consider the figure as shown on page 2.2 in the hydromechanics lecture notes.

1.1 Force

First we calculate the force using $\Sigma F = 0$. We get

$$F = \int_x \int_y \rho g y sin\theta dx dy$$

Which leads to

$$F = \rho g sin\theta \int_x \int_y y dx dy$$

We call the integral the static moment S and we get then

$$S = \int_x \int_y y dx dy$$

or

$$F = \rho g sin\theta S$$

Moreover we can then

$$F = \rho g sin\theta A \frac{\int_x \int_y y dx dy}{A}$$

where

$$y_{cg} = \frac{\int_x \int_y y dx dy}{A}$$

Finally we arrive at the method to calculate very easy the force

$$F = \rho g sin\theta y_{cg} A$$

1.2 Center of pressure

We calculate the resulting moment and use Varignon's theorem. $M = 0$ in two different ways and make the equation of both results to become the center of pressure.

$$y_{cp}F = y\rho g y_{cg} sin\theta A$$

$$y_{cp}F = \int_x \int_y y\rho g y sin\theta dxdy$$

The latter becomes

$$y_{cp}F = \rho g sin\theta \int_x \int_y yydxdy$$

The integral is called the moment of inertia

$$I = \int_x \int_y y^2 dxdy$$

We make the equation of both results (Varignon)

$$y\rho g y_{cg} sin\theta A = \rho g sin\theta I$$

and we get as result

$$y_{cp}S = I$$

And this gives us an easy way to calculate the center of pressure for well known surfaces using tables of static moments and moments of inertia.

1.2.1 Results

We have made the calculations and supposed we were dealing with symetric surfaces. If the surface is not symetric we also have to calculate the x coordinate for the centere of gravity. For this we have to use then the product of inertie called C_{xy}.

$$F = \rho g sin\theta y_{cg} A$$

$$y_{cp} = \frac{I}{S}$$

$$x_{cp} = \frac{C}{S}$$

1.3 Steiner's theorem

If one calculates the moment of inertia they do it around a arbitrary point.(usually it around the center of gravity) But one can also calculate these moments around another point and then you can use the Steiner theorem which says

$$I'_a = I_a \pm {e_a}^2 A$$

where e_a is the distance of translation in the a direction. If you go towards the center of gravity you have to substract(-), if you go away from the center of gravity you have to add up(+).

In our case we use tables where I and C have been calculated for you around the center of gravity so we only have to use

$$I'_a = I_a + {e_a}^2 A$$