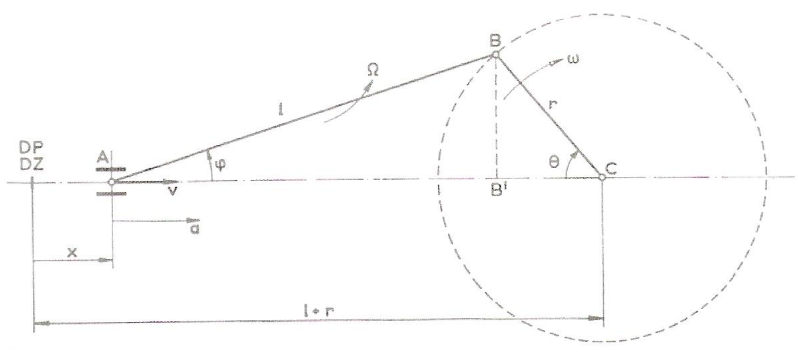
0.1 Crankshaft-connecting rod mechanism



Figuur 1: crankshaft connecting rod mechanism

The goal is to calculate displacement, speed and acceleration of the crankshaft-connecting rod mechanism. The first pumps produced on a large scale were the positive displacement pumps and those positive displacement pump most widely used is the piston pump and these pump are constructed using this mechanism.

If we look at figure 1.1 we see that we can calculate the displacement quite easy. The displacement is x and the total length is l+r. We can distinguish two rectangular triangels,ABB' and CBB'.

$$x = l + r - AB' - B'C$$

We need to calculate AB' and B'C. We need to calculate them in function of time. So as a matter of fact in terms of the rotation speed of the crankshaft which is ω.

Thus we find

$$AB' = l\cos\varphi$$

and

$$B'C = r\cos\theta$$

Now we still need to express φ in function of θ. For this we look at the common side BB'

$$BB' = r\sin\theta = l\sin\varphi$$

. So

$$\sin\varphi = \frac{l}{r}\sin\theta$$

and we set

$$\frac{l}{r} = \lambda$$

0.2

We have to change the sin into a cos and for this we use

$$\cos^2\varphi = 1 - \sin^2\varphi$$

This adds up to

$$\cos\varphi = \sqrt{1 - \lambda^2 \sin^2\theta}$$

Now we get for the displacement of the crankshaft-connecting rod mechanism

$$x = l + r - l\sqrt{1 - \lambda^2 \sin^2\theta} - r\cos\theta$$

This is clearly a non-linear and periodic motion. We can make an approximation by using a Taylor-series for the square root

$$\sqrt{1-x} \approx 1 - \frac{x}{2}$$

In the end we get for the displacement

$$x = l + r - l(1 - \frac{\lambda^2 \sin^2\theta}{2}) - r\cos\theta$$

If one wants to calculate the speed and the acceleration he simply takes respectively the first and the second derivative of the displacement.

We get respectively

$$v = \omega r(\sin(\omega t) + \frac{r}{2l}\sin(2\omega t))$$

$$a = \omega^2 r(cos(\omega t) + \frac{r}{l}\cos(2\omega t))$$

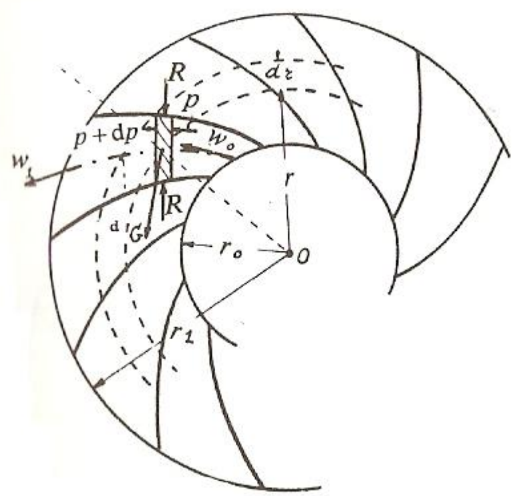
0.2 Increase in pressure caused by a fan

We are going to calculate the increase in pressure caused by the movement of the fan. The pressure is the proportion of the force to the surface.We need to calculate the forces on the fluid that the pump is transporting. We use the principle of D'Alembert

$$\Sigma d\mathbf{F} = \mathbf{a}dm$$

on an infinitely small amount of fluid.

- $\Sigma d\mathbf{F}$:sum of all the external forces working on the body of fluid
- $\mathbf{a}dm$:acceleration of the mass of fluid dm



Figuur 2: forces on an amount of fluid in a fan

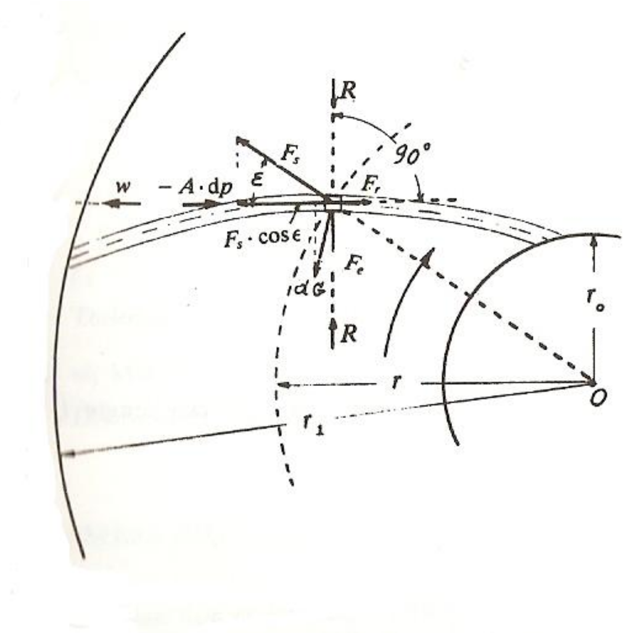
In $\Sigma d\mathbf{F}$ we can find

1. pressure:dp
2. gravitational force:d$\mathbf{G}$
3. reaction on the side of the fan:d$\mathbf{R}$

In the accelerationpart $\mathbf{a}dm$ we can find three components because we are dealing with a relative motion. For the calculation of an acceleration of relative rotational motion we use the theorem of Coriolis.

1. drag acceleration:$\mathbf{a}_d$
2. relative acceleration:$\mathbf{a}_r$
3. Coriolisacceleration:$\mathbf{a}_C$

Now we will look at the diagram of the resulting forces and make the vector sum. We make the sum of the components geprojected on the direction of the relative speed w.



Figuur 3: resulting forces

- **Accelerationforces**
  1. The drag acceleration is centrifugal and the angle between $\mathbf{a}_s$ and w is $\cos\epsilon$
  $$a_s = \frac{u^2}{r} = \omega^2 r$$
  2. The relative acceleration is the change of speed towards the outside and it decreases so this component is negative and pointed to the entry of the fan
  $$a_r = -\frac{dw}{dt}$$

3. The Coriolisacceleration is pointed perpendicular on the surface parallel to O.The projection is equal to zero.

$$a_C = 2\omega w$$

- **Resulting external forces**

1. Pressureforces: These forces are pointed in the direction of w so we get an increase in pressure towards the outlet:-Adp
2. Gravitational force: They are pointed towards the center of the earth. We can say in this case pointed perpedicular downwards. For every small unit of mass there is a counterpart on the other side of the fan so these forces will cancel if we calculate their work(energy).
3. Reaction forces on the fan: They are pointed perpendicular on the side of the fan so the projection on w is zero.

The force is the mass times acceleration. The mass will be calculated using the flow in the fan

$$dm = \rho q dt$$

In the end we get

$$Adp = \rho q dt(\omega^2 r \cos\epsilon - \frac{dw}{dt})$$

The next step is to calculate the change in energy of the flow when it passes through the fan. For this we calculate the displacement dl of the forces intervening on the mass of fluid.

$$(Adp)dl = (\rho q dt)dl(\omega^2 r \cos\epsilon - \frac{dw}{dt})$$

On the other hand we have

$$w = \frac{dl}{dt}$$

or

$$dl = w dt$$

and

$$dl \cos\epsilon = dr$$

We also can use Castelli's law

$$Aw = q$$

We fill these equations in the equation of the force equilibrum

$$Adpwdt = \rho q dt \omega^2 r dr - \rho q dt \frac{dw}{dt} dl$$

$$qdpdt = \rho q dt \omega^2 r dr - \rho q dt w dw$$

$$qdpdt = \rho qdt\omega^2 rdr - \rho qdtwdw$$

$$dp = \rho\omega^2 rdr - \rho wdw$$

This equation has to be integrated over the entire distance from inlet to outlet.

$$\int_{p_0}^{p_1} dp = \rho\omega^2 \int_{r_o}^{r_1} r\,dr - \rho \int_{w_0}^{w_1} w\,dw$$

$$p_1 - p_0 = \frac{\rho}{2}(\omega^2(r_1^2 - r_0^2) - (w_1^2 - w_0^2))$$

Now we have to rewrite the equation and have to consider frictionloss on the fan(skinfriction:p_w).

$$\frac{p_1 - p_0}{\rho g} = \frac{u_1^2 - u_0^2}{2g} + \frac{w_0^2 - w_1^2}{2g} - \frac{p_w}{\rho g}$$

0.2.1 Calculation of the work by a fan

We use Bernouilli's law between intake and outlet of the fan.

$$\frac{p_0}{\rho g} + \frac{c_0^2}{2g} = \frac{p_1}{\rho g} + \frac{c_1^2}{2g} + W'$$

From this we get

$$W = \frac{p_1 - p_0}{\rho} + \frac{c_1^2 - c_0^2}{2}$$

Again we need to consider frictional losses so we get

$$W = \frac{p_1 - p_0}{\rho} + \frac{c_1^2 - c_0^2}{2} + \frac{p_w}{\rho}$$

We can now replace the pressureterms and finally become

$$W = \frac{c_1^2 - c_0^2}{2} + \frac{u_1^2 - u_0^2}{2} + \frac{w_0^2 - w_1^2}{2}$$

Now we simply use some basic trigonometry on the vectordiagram of intake (i=0) and outlet(i=1)

$$w_i^2 = c_i^2 + u_i^2 - 2c_i u_i \cos\alpha_i$$

and we get to the formula for the work

$$W = u_1 c_1 \cos\alpha_1 - u_0 c_0 \cos\alpha_0$$